Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number

Candidate Number

Monday 21 January 2019

Morning (Time: 2 hours)	Paper Reference **4PM0/02**

Further Pure Mathematics
Paper 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over

Pearson

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Solve the equation $3\log_3 x - 8\log_x 3 = 10$

(6)

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Question 1 continued

(Total for Question 1 is 6 marks)

2 (a) Using the axes below, sketch the line with equation

(i) $y + 2x = -5$ (ii) $y = x + 4$

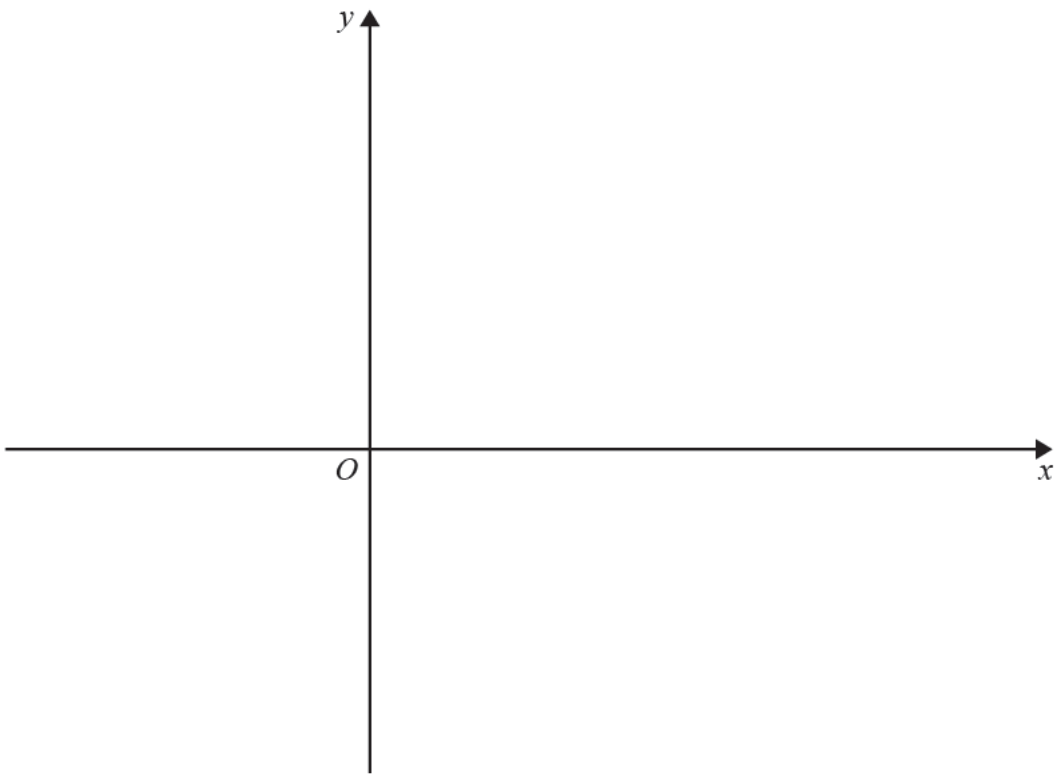
Show the coordinates of the points where each line crosses the coordinate axes.

(2)

(b) Show, by shading, the region R defined by the inequalities

$y + 2x > -5$ $y < x + 4$ $x < 1$

(1)



Question 2 continued

(Total for Question 2 is 3 marks)

3 Referred to a fixed origin O, the position vectors of the points P and Q are $(5\mathbf{i} + 6\mathbf{j})$ and $(3\mathbf{i} - 4\mathbf{j})$ respectively.

(a) Find, as a simplified expression in terms of $\mathbf{i}$ and $\mathbf{j}$, $\overrightarrow{PQ}$.

(2)

(b) Find a unit vector parallel to $\overrightarrow{PQ}$.

(2)

The position vector of the fixed point R is $(13\mathbf{i} + a\mathbf{j})$, where a is a constant.

Given that $\overrightarrow{QR} = 5\overrightarrow{QP}$

(c) find the value of a.

(2)

Question 3 continued

(Total for Question 3 is 6 marks)

4 A particle P is moving along the x-axis. At time t seconds ($t \geqslant 0$) the velocity, v m/s, of P is given by $v = 4\sin 2t$

(a) Find the least value of t for which the velocity of P is 2 m/s.

(2)

(b) Find the magnitude of the acceleration of P when its velocity is 2 m/s.

(3)

The particle P is at the point with coordinates (3, 0) when $t = \dfrac{\pi}{4}$

(c) Find the distance of P from the origin when $t = 0$

(4)

Question 4 continued

(Total for Question 4 is 9 marks)

5

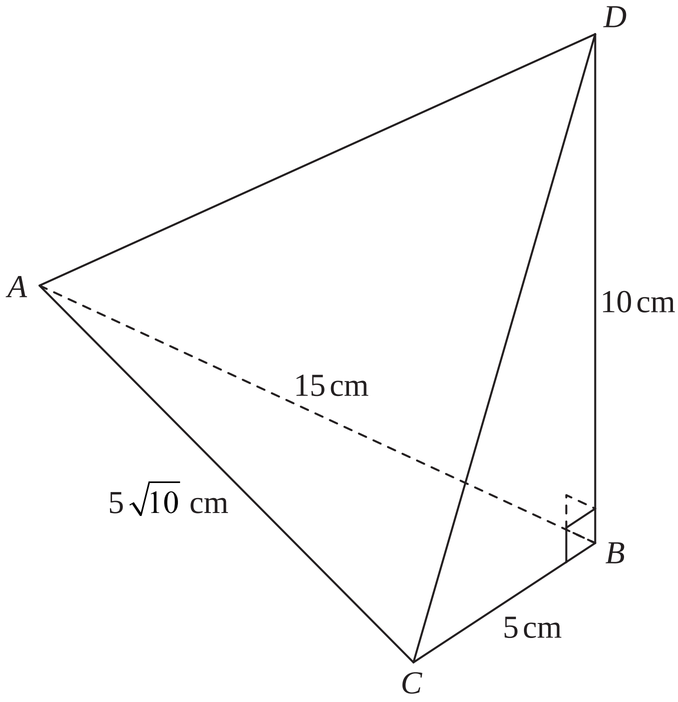


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a triangular pyramid *ABCD* where triangle *ABC* is the base and *BD* is perpendicular to the base.

$$AB = 15\text{ cm} \quad AC = 5\sqrt{10}\text{ cm} \quad BC = 5\text{ cm} \quad BD = 10\text{ cm}$$

(a) Show that $\angle ABC = 90°$

(2)

(b) Find, in degrees to 1 decimal place, the size of $\angle DAC$.

(4)

The point *X* on *AC* is such that *BX* is perpendicular to *AC*.

(c) Find, in degrees to 1 decimal place, the size of $\angle DXB$.

(4)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6

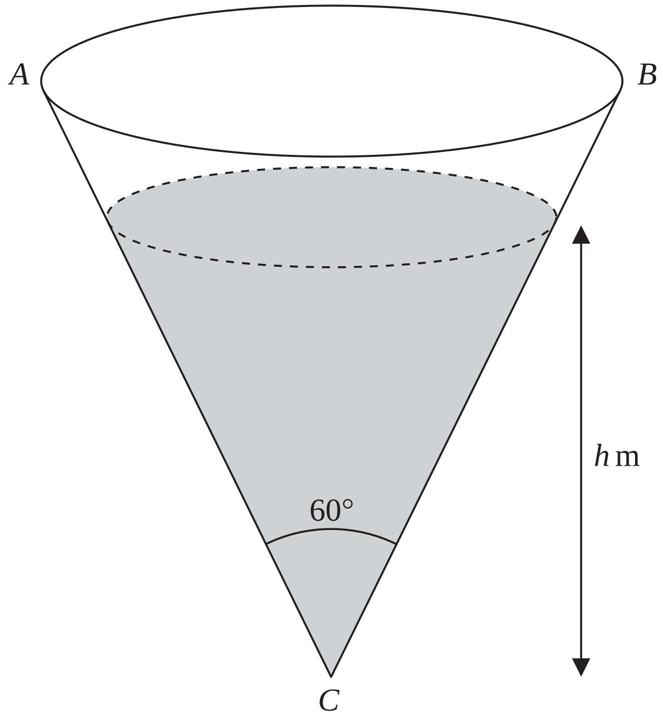


Figure 2

Figure 2 shows a water tank in the shape of a hollow right circular cone fixed with its axis of symmetry vertical. A diameter of the circular rim of the cone is AB. The vertex, C, of the cone is below AB such that $\angle ACB = 60°$

Initially, the tank is empty and water flows into the tank at a constant rate of $0.03\,\text{m}^3/\text{s}$.
At time t seconds after the water starts to flow into the tank, the height of the surface of the water in the tank above C is h metres.

Find, in m/s to 3 significant figures, the rate of change of the height of the surface of the water above C at the instant when $h = 1.5$

(6)

Question 6 continued

(Total for Question 6 is 6 marks)

7 (a) Complete the table of values for $y = \ln(3x + 1) + 2$, giving your answers to 2 decimal places.

x	0	1	2	3	4	5	6
y	2		3.95	4.30			4.94

(2)

(b) On the grid opposite, draw the graph of $y = \ln(3x + 1) + 2$ for $0 \leqslant x \leqslant 6$

(2)

(c) Use your graph to obtain an estimate, to 1 decimal place, for the value of $\ln 10.6$
You **must** show clearly how you have used your graph.

(3)

(d) By drawing a straight line on the grid, obtain estimates, to 1 decimal place, for the roots of the equation $(3x + 1)^2 = e^{(x + 1)}$ in the interval $0 \leqslant x \leqslant 6$

(5)

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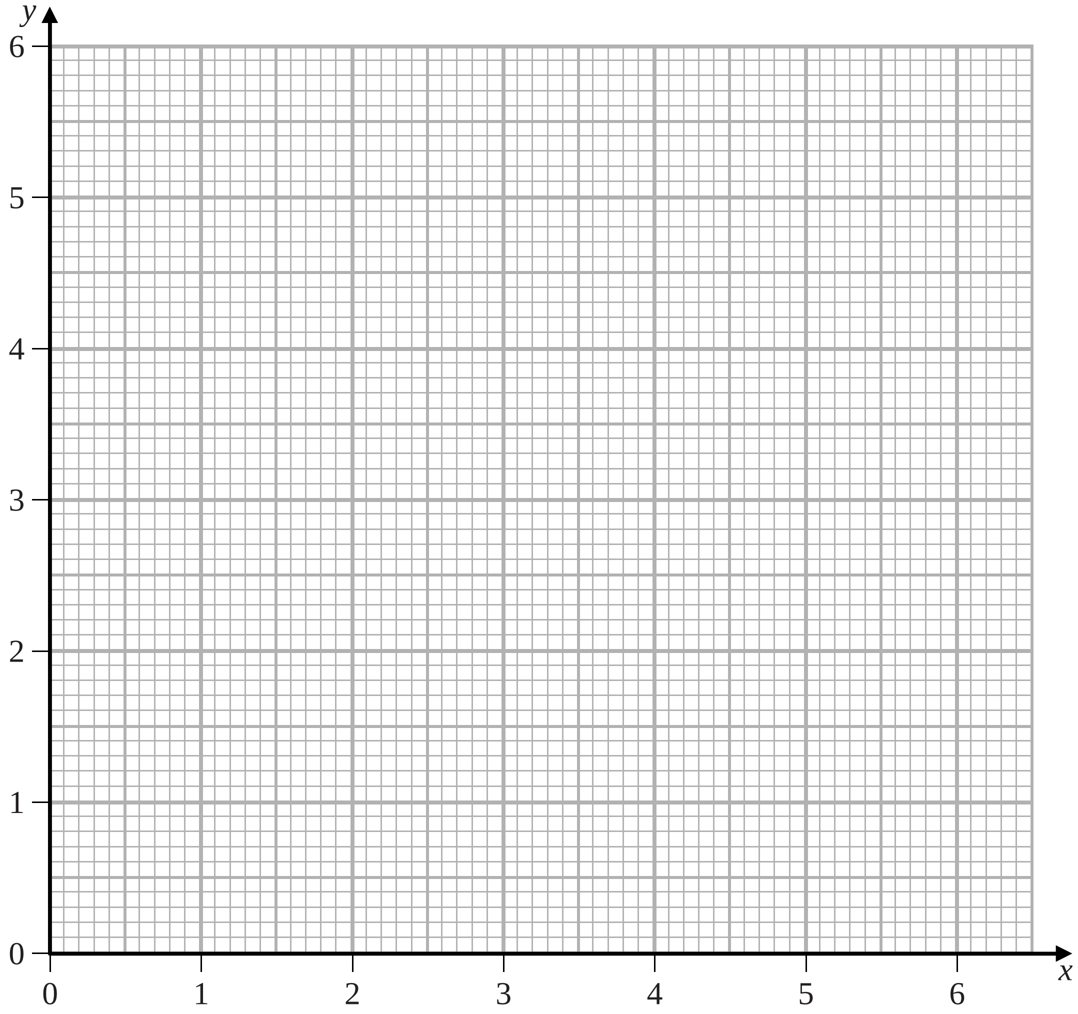
Question 7 continued



Turn over for a spare grid if you need to redraw your graph.

Question 7 continued

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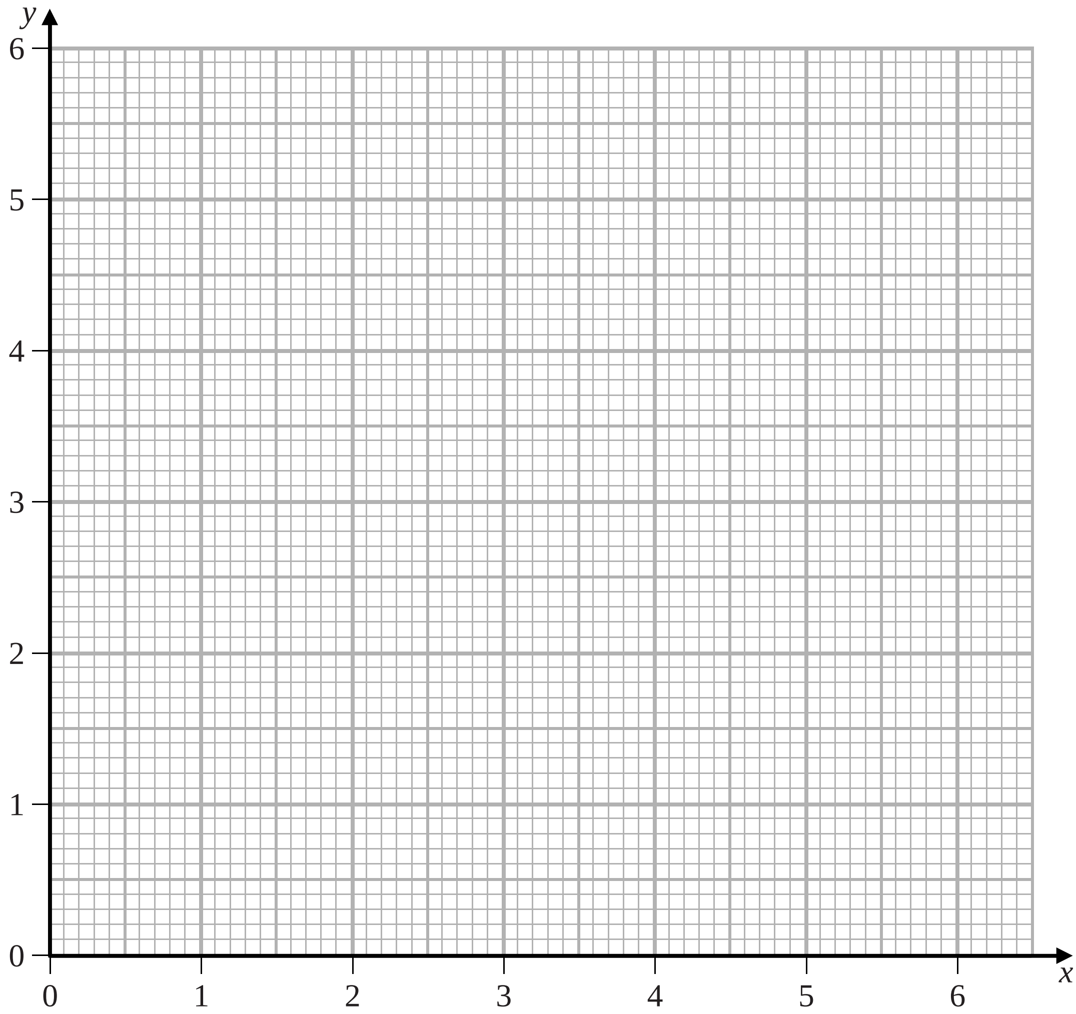
Question 7 continued

Only use this grid if you need to redraw your graph.



(Total for Question 7 is 12 marks)

8 The roots of the equation $3x^2 - 2x - 1 = 0$ are α and β, where $\alpha > \beta$

Without solving the equation,

(a) find the value of $\alpha^2 + \beta^2$

(3)

(b) show that $\alpha - \beta = \frac{4}{3}$

(2)

(c) form a quadratic equation, with integer coefficients, that has roots $\frac{\alpha + \beta}{\alpha}$ and $\frac{\alpha - \beta}{\beta}$

(6)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 11 marks)

9

Diagram **NOT** accurately drawn

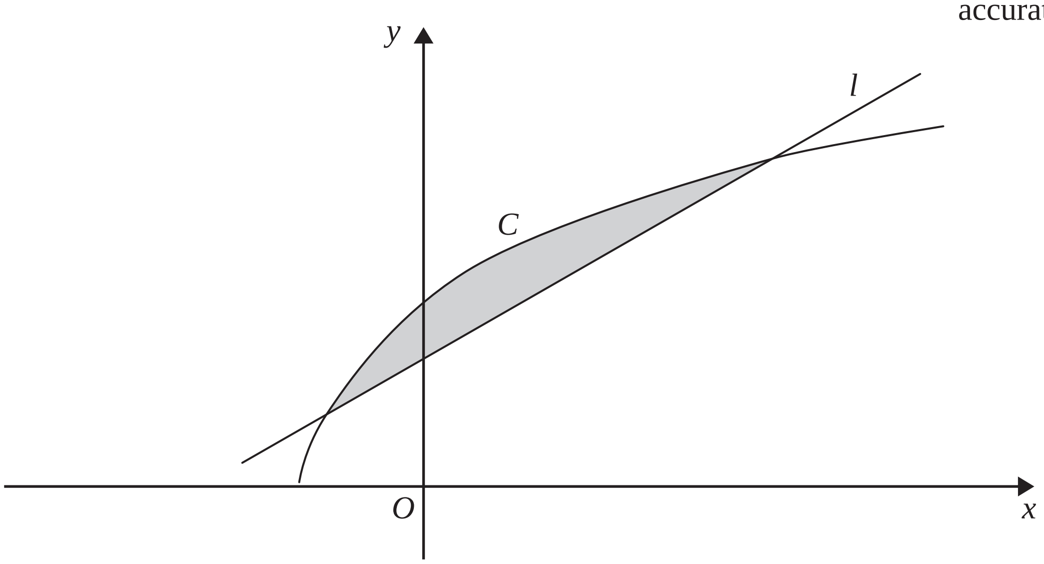


Figure 3

Figure 3 shows part of the curve C with equation $y = (2x + 3)^{\frac{1}{2}}$ and the line l with equation $2y = x + 3$

The line l crosses C at two points.

(a) Find the coordinates of each of these points.

(5)

The finite region bounded by C and l, shown shaded in Figure 3, is rotated through 360° about the x-axis.

(b) Use algebraic integration to find, in terms of π, the volume of the solid generated.

(5)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

10 A geometric series has first term a and common ratio r $(r > 0)$
The nth term of the series is U_n

Given that $U_1 + 3U_2 = 8$ and that $U_2 \times U_3 = 4U_5$

(a) find

(i) the value of r

(ii) the value of a

(5)

(b) Hence show that $U_n = \dfrac{2^{n+2}}{3^n}$

(2)

(c) Find the least value of n such that $U_n < 0.05$

(3)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

11

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

(a) (i) Using the above identity, show that

$$\cos 2x = 1 - 2\sin^2 x$$

(ii) Hence show that

$$\frac{13\sin x - 2\cos 2x - 10}{4\sin x - 3} = 4 + \sin x$$

(7)

(b) Hence solve, in radians to 3 significant figures, the equation

$$10 + 2\cos\left(2\theta + \frac{\pi}{3}\right) - 13\sin\left(\theta + \frac{\pi}{6}\right) = 2\sin\left(\theta + \frac{\pi}{6}\right) + 8$$

for $\pi \leqslant \theta \leqslant 2\pi$

(5)

(c) Find the exact value of

$$\int_0^{\frac{\pi}{2}} \left(\frac{13\sin x - 2\cos 2x - 10 + 4x\sin x - 3x}{4\sin x - 3}\right) dx$$

(5)

Question 11 continued

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 17 marks)

TOTAL FOR PAPER IS 100 MARKS